Dynamic Linking of Multiple Representations

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Abstract. Virtual manipulatives (VM's) are free, online, interactive objects, usually java applets that often include multiple linked representations. This linking can be dynamic (automatic) or constructed (by the user). In this study, we investigate the hypothesis that constructing representations may increase opportunities for learners to develop conceptual connections between the representations, promoting transfer of understanding of mathematics topics. In comparing 9 year olds' performance on fraction tasks in a dynamic and a construct condition, we find that while both conditions lead to growth in performance, students in the construct condition show more conceptual connections between the representations.

Keywords: virtual manipulatives; multiple representations; learning; mathematics education.

Introduction

Virtual manipulatives (VM's) are a resource to help students develop conceptual understanding in mathematics. VM's are free, online, interactive objects, usually java applets, similar to physical manipulatives (PM's) frequently used in mathematics classrooms. VM's can easily display multiple representations. For example, Figure 1 shows a VM for fractions with two representations—1/5 shaded in on a shape and the written fraction 1/5.

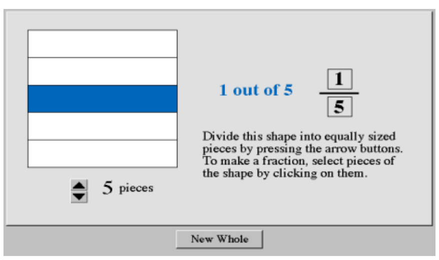

Figure 1. Fraction VM

These two representations can be connected through dynamic linking or constructed linking. By dynamic linking, we mean that when one representation changes, the other changes automatically (e.g., a user types in 2/5 in the VM in Figure 1 and 2 blocks are automatically shaded). By constructed linking, we mean that the user creates the second representation (e.g., the VM shows 2/5 and the user shades in the blocks to make the fraction).

There is significant research on the effects of multiple representations[1]. However, there is less on the effects of different types of linking. Current research has focused primarily on how dynamically linking representations may benefit learners when compared with using separate, unlinked representations. This study investigates the effects of learners constructing representations for fractions.

We hypothesize that constructed linking of representations may increase opportunities for learners to

[1] Due to space restrictions, we are not including references in this abstract. We will include them in the full paper.

develop conceptual connections between the representations. Developing these connections may lead to better transfer of understanding of mathematics topics.

Methods

Thirty-five nine-year-olds (third year in elementary school) were randomly assigned to the dynamic (N = 18) or the construct (N = 17) condition. Students in the dynamic condition used the VM shown in Figure 1. Students in the construct condition used a variant of this VM that hid the linked number representation (See Figure 2). These students constructed that representation. All students took a pre- and posttest with 3 learning sessions (interviews) conducted in between.

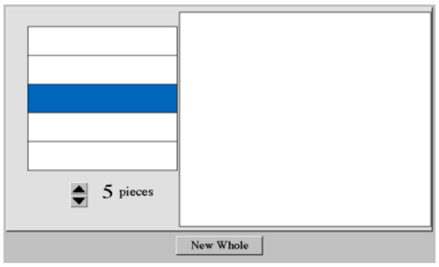

Figure 2. VM modified for construct condition

In each of the 3 learning sessions, students solved 12 problems like those shown in Figure 3 and received graduated feedback. Students created the fraction using the VM, the first representation, and then wrote the fraction answer on a separate sheet of paper, the second representation. Students in the dynamic condition saw the second representation on the VM and could simply copy it on their paper, while students in the construct condition created the second representation on their own. The pre and posttest each had 9 domain problems like the learning sessions and 6 transfer problems covering addition with symbols or pictures and fraction comparison.

	Questions 1-4	Questions 1-4	Questions 1-4
Task	Create an equivalent fraction given a different denominator	Create a fraction equivalent to a given fraction	Create a second fraction equivalent to the given fraction
Example	Given: A rectangle divided into 8 pieces Question: Can you show me one fourth of this rectangle?	Given: A rectangle divided into 6 pieces Question: Can you make a fraction that is equivalent to two thirds?	Given: A rectangle divided into 6 pieces Question: Can you make a different fraction that is equivalent to two thirds?
Possible Student Response	 8 pieces	 6 pieces	 9 pieces

Figure 3. Sample problems from learning sessions

Analysis

Accuracy

For the pre and posttests, accuracy was the total number of problems answered correctly. For the learning interviews, accuracy was the number of problems answered correctly on the first attempt (i.e., without feedback).

Conceptual Connections

Students' written fraction representations were in reduced form (e.g., 6/8 of the rectangle shaded, student writes 3/4) or non-reduced form (e.g., 6/8 of the rectangle shaded, student writes 6/8). We take it that a student who writes the reduced form of the fraction has made a deeper conceptual connection between the representations. Rather than simply copying a fraction like 6/8 exactly, the student is transferring to the more challenging equivalent fraction, 3/4.

Results and Conclusions

Accuracy

While all students learned over the course of the learning sessions, there were no significant differences between conditions. The main effect of interview time was significant, $F(2, 66) = 36.75$, $p < .01$, but the interaction between interview time and condition was not significant, $F(2, 66) = 0.331$, $p > .05$. In other research on multiple representations, students given explicit connections between representations often outperform students who are not. While further research is necessary, this result suggests instead that VM's that prompt students to construct multiple representations may be as beneficial to students as VM's with dynamically linked representations. While students in both conditions showed no differences in accuracy during the learning sessions, there were differences in the conceptual connections made.

Conceptual Connections

As expected, students in the construct condition wrote fractions in reduced form more often ($M = 9.41$, $SE = 3.02$) than those in the dynamic condition ($M = 2.17$, $SE = 1.70$). We contend that construct students reconciled the numerical representation they wrote with the unreduced pictorial representation they created, showing a deeper understanding of equivalent fractions. However, construct students may simply be remembering the interviewer's question, which contained the name of the reduced fraction. Further research is needed to determine if this is the case. However, our interactions with students suggest that it is unlikely. All students demonstrated that they understood that a fraction can be expressed symbolically by writing the total number of pieces in the denominator and the number of pieces shaded in the numerator (i.e. the unreduced form), so when they wrote a written fraction in a reduced form, it seems likely that they understood that both fractions were equivalent.

Discussion

The results of this study suggest that constructing representations can prompt students to make more connections and develop deep understanding. Constructing representations can prompt students to think deeply about the way the content is represented allowing them to make more connections and develop deep understanding. As many mathematics VM's with multiple representations employ dynamic linking, these results open the door for new designs of VM's. We will further investigate the ways constructing representations, perhaps even combined with dynamic linking, supports student learning.