Determinants of Successful and Non-Successful Solutions of Complex Mathematical Word Problems

Katharina Hohn[1], Johannes Groß[1], Sibel Telli[1], Wolfgang Schnotz[2] & Renate Rasch[3]
hohn@uni-landau.de, gross@uni-landau.de, telli@uni-landau.de,
schnotz@uni-landau.de, r-rasch@uni-landau.de
[1] University of Koblenz-Landau, DFG-Graduate School "Teaching Processes",
Thomas-Nast-Straße 44, 76829 Landau, Germany,
[2] University of Koblenz-Landau, Department of General and Educational Psychology,
Fortstraße 7, 76829 Landau, Germany
[3] University of Koblenz-Landau, Institute of Mathematics,
Fortstraße 7, 76829 Landau, Germany

Abstract. Solving mathematical word problems is a required ability in each grade level. Different solution strategies are taught, but assessments of mathematical ability could not demonstrate the desired achievements. Since the processes of problem solving are not sufficiently explored, the development of more adequate teaching methods is difficult. Thus, this project focuses on the processes of mathematical problem solving in primary school with reference to intrapersonal factors that are significant to the mathematical achievement. Performances of children from 2nd and 4th grade were explored while solving complex mathematical word problems. Results indicate that all students had major difficulties. Students from 4th grade reached a higher number of correct solutions. Moreover, their processes of problem solving were better structured and they used more external representations than students from 2nd grade. The influence of further factors is discussed.

Keywords: Representations; Problem Solving; Mathematical Education; Primary School

Theoretical background

Graphics and texts have always been essential elements in educational contexts. They are introduced at the very beginning of school and gradually become more important. One aspect within this context is the solution of mathematical word problems. Handling them is already a part of primary school curriculum, but international and national assessments of mathematical ability have shown that successful solutions of word problems are seldom (e.g. Baumert, Lehmann & Lehrke, 1997). Despite the rich literature on word problems, the processes of problem solving are rarely known (Verschaffel, De Corte & Vierstraete, 1999), especially in primary school. Thus, the following research project focuses on a deeper understanding of students' processes of mathematical problem solving with regard to complex word problems. The latter are defined as tasks, based on challenging mathematical structures, which are embedded in settings, so that familiar calculations are not obvious offhandedly, respectively cannot be applied without any transformational effort (cf. Rasch, 2001, p. 26).

Although problem solving is usually regarded as a directional process (cf. Polya, 1995), primary school students seem to solve word problems in a less structured manner (Rasch, 2001).

Furthermore, solving complex word problems implicates a multitude of cognitive operations leading to increased working memory requirements. By creating an external representation, such as drawings, tables or equations, the individual deposits some of those requirements to the environment (Schnotz, Baadte, Müller & Rasch, in press). Such behavior offers an additional opportunity to have a more elaborated look on the process of problem solving. So, external representations were examined and compared across two different grade levels of primary school students.

According to previous research, variables like verbal and visual/spatial intelligence, self-efficacy and mathematical self-concept have been taken into account since they are considered to be important determinants of students' mathematical ability (e.g. Pajares & Miller, 1994; Stock, Desoete & Roeyers, 2009; Marsh & Yeung, 1997).

In summary, the present study aimed to explore the processes of mathematical problem solving in a sample of primary school students and investigated the following research questions.

Research questions

- Which individual prerequisites of students are required for solving different kinds of mathematical word problems successfully?
- How are differences in problem representations related to success or failure of problem solving activities?
- How do representational skills regarding mathematical problem solving develop with grade level?

Method

Students from each one class of the 2nd and the 4th grade participated in the study; data were gathered in one German primary school. The whole study was conducted by two previously instructed researchers with whom the students were not familiar.

Firstly, verbal and visual/spatial intelligence were measured by scales from a cognitive abilities test (Kognitiver Fähigkeitstest [KFT], Heller & Geisler, 1983; Heller, Schön-Gaedike & Weinläder, 1976). Different test versions were applied with regard to the two different grade levels. Additionally, scales from the Project for the Analysis of Learning and Achievement in Mathematics (Projekt zur Analyse der Leistungsentwicklung in Mathematik [PALMA], Pekrun, Götz, Zirngibl, v. Hofe & Blum, 2002) were used to assess students' mathematical self-efficacy as well as their mathematical self-concept.
Afterwards they were asked to solve individually five different complex mathematical word problems, such as "Two bandits discover a hidden treasure, 2 bags of gold coins. They count the coins. In one bag there are 34 coins, in the other there are 52 coins. They want to share the prey fairly. How many coins do they have to take off the fuller bag and put in the other bag, until the number of coins is equally distributed?" (cf. Rasch, 2008). All word problems were taken from a task selection, which was used in former research (Rasch, 2001).

The word problems were read out by the researcher and then given to each child in written form. In order to grow accustomed to the whole procedure, the same word problem was presented as a first task to all students. The order of the following tasks was chosen with regard to a Latin Square Design to avoid sequence effects. Students were allowed to use different auxiliary materials (sheet of paper, colorful pencils, 1-unit cubes, 10-unit rods). They were videotaped and interviewed individually right after every task via a semi-structured interview with regard to their solution process, thoughts and ideas.

Spontaneous processes of problem solving were examined as well as the used forms of representations. This happens on the basis of the notes of the students (if available) and video data. In this connection independent raters also assessed motivation and interest of the children with regard to the processing of the single word problems. Hence, the influence of the context of the different word problems on aspects of the solution process could be taken into account. Furthermore data from the cognitive abilities test as well as data from mathematical self-efficacy and self-concept questionnaire were analyzed. Thereby, a structural system was developed that offers the possibility to understand what combinations of personal and procedural factors lead to successful solutions.

Preliminary Results

35 students from 2nd and 4th grade of one German primary school participated in the study. 19 students were from 2nd grade (Mage=8.53, SD=.61), 10 of them were girls. The other 16 students attended the 4th grade (Mage=10.31, SD=.60), 9 of them were girls.

Analysis of the videos revealed that students of both grade levels had difficulties in solving the complex word problems. The strategies of problem solving differed with regard to the content of the single word problems. Depending on grade level, different forms of representations were used that had an impact on the quality of the process of solution as well as the needed time. External representations were significantly more often used by students from the 4th grade. Furthermore, grade level had an effect on the number of correct solutions and the structure of the process of problem solving.

The influence of further factors (e.g. gender, verbal and visual/spatial intelligence, self-efficacy, mathematical self-concept) is discussed.

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