# Blocked versus Interleaved Practice with Multiple Graphical Representations of Fractions

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**Abstract.** Prior research shows that multiple representations can enhance students' learning. In order to learn with multiple representations, students need to acquire representational fluency with each of the representations, as well as representational fluency. It is yet unclear how to balance these two aspects of learning with multiple representations. In the present study, we focus on a key aspect of this question, namely the temporal sequencing of representations presented one-at-a-time. Specifically, we investigated the effects of blocking vs. interleaving multiple graphical representations of fractions in an online tutoring system. We conducted an *in vivo* experiment with 296 5th- and 6th-graders. Results show an advantage for blocking representations and an increasingly interleaved sequence, suggesting that representational fluency enables flexibility more so than the other way around.

**Keywords:** Multiple representations; fractions; blocked vs. interleaved practice; classroom evaluation

## Introduction

Understanding fractions is foundational for more advanced mathematics (NMAP, 2008), yet fractions pose a significant challenge for students. In an earlier study we found that students working with multiple graphical representations of fractions learn better than students working with a single graphical representation, when prompted to explain how the graphical representations (e.g., a circle) relate to the symbolic representation (e.g., 1/2) (Rau, Aleven, & Rummel, 2009). This finding is in line with a number of studies that demonstrate benefits for learning with multiple representations (Ainsworth, Bibby, & Wood, 1998). However, providing students with multiple representations is not always beneficial (see Ainsworth, 2006), which has been attributed to the fact that they require learners to acquire several cognitive competencies: Learners need to understand the particular representations and to use them appropriately; in other words, they need to acquire *representational fluency* with each representation (Ainsworth, 2006). In addition, students can only benefit from learning with multiple representations if they are able to make comparisons across representations and translate between them; in other words, they need to develop *representational flexibility* (Spiro & Jehng, 1990).

At this point, it is an open question how to balance the support of representational fluency and representational flexibility in order to maximize students' learning. In the present study, we consider the *temporal sequencing* of multiple graphical representations presented one-at-a-time. Specifically, we contrast blocking representations (e.g., AAABBBCCC, where A may be a circle, B a numberline, and C a set representation), versus interleaving them (e.g., ABCABCABC). When practice with representations is blocked, students have the opportunity to develop fluency with one representation before the next one is introduced. On the other hand, when practice with different representations is interleaved, students may have greater opportunity to (spontaneously) make comparisons between representations and develop representational flexibility along with fluency. If the development of representational flexibility relies on students' fluency with the individual representations, students should learn best when practice with the representations is blocked. If, on the other hand, representational flexibility can develop along with or even contribute to students' acquisition of representational fluency, students should learn best when practice with the representations is interleaved. We think that making comparisons between representations without having acquired fluency with each of them will increase cognitive load. We therefore hypothesize that students' benefit from the opportunity for comparison making builds on fluency, and thus predict an advantage for a condition where students first learn with each representation in a blocked fashion and then gradually shift into a mode where they encounter the different representations in an interleaved fashion (i.e. the frequency of switching is increased).

**Methods**

*Experimental Design and Procedure*

We investigated our research question in the context of an intelligent tutoring system for fractions learning, which we are building concurrent with our studies. A total of 269 5th- and 6th-graders worked on the fractions tutor for 5 hours during class time. We assessed students' understanding of fractions representations and of operations with fractions before, directly after, and one week after the intervention. We assigned students randomly to one of four conditions: blocked, moderate, interleaved, and increased. Students in all conditions worked on the same 102 problems. Figure 1 clarifies how the conditions were implemented. Each table represents the set of 102 problems that students solved with the tutor. Each row represents a topic (e.g., equivalent fractions, or fraction addition). (We counterbalanced different plausible orders of representations in order to prevent possible order effects, so that the table represents only one of the possible orders of representations.) In the blocked condition, students switch representations after having worked on all circle problems (corresponding to 36 problems per representation). Students in the moderate condition work on all circle problems of topic 1, corresponding to three problems per representation before switching. In the interleaved condition, students switch representations after each single problem. Finally, in the increased condition, the sequence of problems corresponds to the blocked condition for topics 1 and 2, to the moderate condition for topics 3 and 4, and to the interleaved condition for topics 5 and 6.

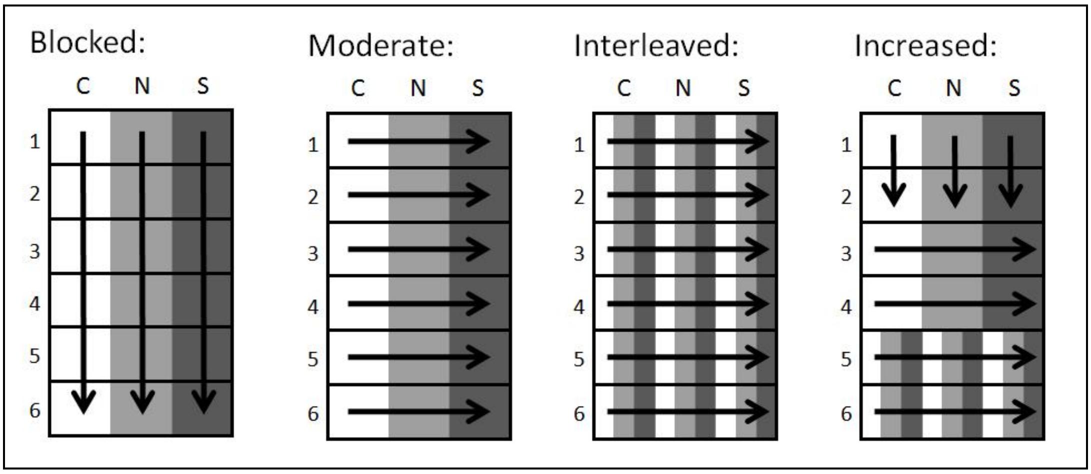


*Figure 1. Rows depict six different topics covered, columns shows one possible order of representations: C/white = circle, N/light-grey = numberline, S/dark-grey = set.*

*Materials*

The tutoring system included three different graphical representations of fractions: circles, numberlines, and sets. We assessed students' knowledge of fractions three times: on the first day of the study, on the day following the tutor sessions, and seven days after the end of the tutor sessions.

Students' understanding of fractions was assessed on representational knowledge and operational knowledge. By representational knowledge, we mean the ability to interpret representations of fractions. Operational knowledge describes the ability to solve fractional tasks.

**Results**

Students who were present for all test days were included in the analysis, yielding a total of $N$ = 215. The reported $p$-values are adjusted using the Bonferroni correction.

We had predicted an advantage for the increased condition at the immediate and a the delayed post-test. The results partly support this hypothesis. We found a significant interaction effect between test time and blocking, for representational knowledge, $F(6, 422) = 5.54$, $p < .01$, and operational knowledge, $F(6, 422) = 2.19$, $p < .05$. Post-hoc comparisons showed that regarding representational knowledge, the blocked condition significantly outperformed the interleaved condition at the immediate post-test ($p < .05$). At the delayed post-test, both the blocked and the increased conditions performed significantly better than the interleaved and moderate conditions ($p$s < .01). Post-hoc comparisons did not reveal statistically significant differences on operational knowledge.

**Discussion and Conclusion**

Our results show an advantage for the blocked and increased conditions and thus support the notion that students' acquisition of representational fluency is prerequisite to their ability to benefit from opportunities to make cross-representational comparisons. At the level of cognitive processes, this supports the idea that representational fluency facilitates the development of representational flexibility. Our findings stand in contrast to earlier findings which demonstrate an advantage for interleaved practice over blocked practice (de Croock & Van Merrienboer, 1998). The difference between our studies and prior research is that we investigate the effects of blocked versus interleaved practice with graphical representations as opposed to blocked versus interleaved practice of different problem types.

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