Representational Flexibility in Linear Function Problems

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Abstract. This study evaluated students' representational flexibility in linear function problems. Eighty-six secondary-school students solved problems under one choice condition, where they chose a table, a formula, or both to solve each problem, and two no-choice conditions, where one representation was forced upon them. Two approaches to the evaluation of flexibility were used : a task-based approach, where students were said to make a flexible choice if they selected the representation that better matched the problem at hand, and a task × student-based approach, where students' choices were considered flexible if they selected the representation that, according to their no-choice data, was the most likely to help them reach the solution. A strong correlation between task × student-based flexibility and choice condition performance was found, which suggests that students who make flexible representational choices are more likely to succeed in problems involving a representational choice than students whose choices are inflexible.

Keywords: Multiple external representations; choice; flexibility; adaptivity; problem solving; mathematics; linear functions.

Theoretical Background and Justification of the Study

In the last decades, the role of external representations in school mathematics has become increasingly important (Monk, 2003). Often, technology is used to create multi-representational environments that expose students to multiple external representations (MERs) of the same concepts with the hope that such exposure will improve their problem-solving performance. It has been claimed that, for students to benefit from MERs in mathematical tasks, they need to possess two main skills:

1. Representational fluency, which means having the necessary diagrammatic knowledge to interact with the representations (Roth & Bowen, 2001), being able to interpret representations by linking them with reality (Ainsworth, Bibby, & Wood, 1998), and mastering the skills of translating and switching between representations within the same domain (Even, 1998).

2. Representational flexibility or adaptivity[1], which involves making appropriate representational choices. The way in which flexibility is conceptualized varies across studies. In some studies (e.g.

[1] For the sake of simplicity, we use the terms flexibility and adaptivity as synonyms in this contribution, although not all authors would do this.

Schnotz & Bannert, 2003), a choice is considered flexible if it is tuned to the demands of the task at hand. In other studies, especially from the strategy choice literature (e.g., Verschaffel, Torbeyns, De Smedt, Luwel, & Van Dooren, 2007), a choice is considered flexible if it matches not only task demands, but also the characteristics of the subject making the choice. There is extensive research on representational fluency (e.g. Even, 1998; Roth & Bowen, 2001). However, research on representational flexibility is scarce. This study focused on representational flexibility in mathematical problem solving. Our hypothesis was: Representational flexibility is a characteristic of efficient mathematical problem solving in the sense that students who display such flexibility are likely to perform better than students who display very low or no flexibility.

Method

Eighty-six secondary-school students from ninth, tenth, and eleventh grade participated in the study. They all had prior knowledge about linear functions and were familiar with the two kinds of representations (tables and formulas) used in this study.

All students solved a paper-and-pencil test consisting of contextualized problems which required finding values within a function or finding the intersection point between two functions. The choice/no-choice (C/NC) method (Siegler & Lemaire, 1997) was used. In the C condition, students chose a table, a formula, or both to solve each problem. In the NC table and NC formula conditions, students could only use the representation provided. The C and NC table conditions included read-off problems, where the answer was displayed in the given table(s); interpolation problems, where the answer was located between two of the points in the table(s); close prediction problems, where the answer was one step away from the data in the table(s), and far prediction problems, where the answer was several steps away from the data in the table(s). The functions in the NC formula condition had slopes and intercepts comparable to those in the NC table and C conditions. The C condition was administered first, followed by the NC conditions some days later. Half of the students were exposed to the NC table condition first, whereas the other half were exposed to the NC formula condition first. Students were instructed to show their work and not to use a calculator.

Summary of Results

Two approaches to the evaluation of flexibility were used: a task-based approach, and a task × student-based approach. When the task-based approach was used, students' choices were considered flexible if they selected the representation that, according to a rational task analysis, was the best match for the problem at hand. This approach revealed that, although there was a strong tendency for students to choose the formula across problem types, some students' choices were affected by the characteristics of the to-be-solved problems in the direction predicted by our rational task analysis. However, the task-based approach did not allow us to determine whether the students who, for example, flexibly switched from the table to the formula from RO to IP intersection problems were the same ones who displayed (task-based) flexibility by also using the formula in CP and FP problems.

The task × student-based approach yielded a more individualised view of representational flexibility. Flexibility was assessed by comparing a student's choices in the C condition for each problem with his performance in the parallel problem in the NC table and NC formula conditions. A choice in a problem in the C condition was considered flexible (and thus given a flexibility score of 1 for that problem) if the student chose a representation with which he successfully solved a parallel problem in the corresponding NC condition. Conversely, a student was given a –1 flexibility score if he chose a representation with which he was unable to solve a parallel problem in the corresponding NC condition, while he solved the problem correctly with the other representation in the other NC condition. The analysis of the mean flexibility scores for each student showed that, for many students, sticking to the formula across problem types was actually a flexible choice, since these students were able to solve most of the problems in the NC formula condition but not in the NC table condition. Strong correlations (va lue problems $r = .60$, $p <.001$; intersection problems $r = .83$, $p <.001$) between the task × student-based flexibility score and C performance confirmed our hypothesis – representational flexibility indeed seems to be a characteristic of efficient mathematical problem solving.

Conclusion

This experiment provided evidence to support our hypothesis that, in the context on linear function problems, representational flexibility is a characteristic of efficient problem solving. Research effort needs to be invested in designing powerful instructional tools that pay explicit attention to the development of students' representational flexibility besides their representational fluency.

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