Children's Spontaneous Use of Different Representations and Further Elements of the Problem Solving Process on Complex Story Problems

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Abstract The aim of this study was the examination and analysis of the spontaneous use of different representations and further elements of the problem solving process on complex story problems at primary schools. A total number of 18 students of 2nd and 20 students of 4th grade participated in our study. According to their results in a German standardized mathematics proficiency test, they were divided into the groups of "higher maths competence" and "lower maths competence". Our findings show that the ability groups and the class levels differed in terms of representations they used, as well as in the time they needed to solve the problems. Also, the total word problem score differed in class levels and ability groups.

Keywords: Use of Different Representations; Problem Solving Process; Complex Story Problem

Theoretical Background

The term "complex story problems" specifies a group of tasks that are different from "regular story problems". Unlike regular story problems, these tasks are based on very challenging mathematical structures and cannot be solved by arithmetic operation models, students are usually familiar with. In order to solve these specific tasks, students have to restructure their prior knowledge (Winter, 1992).

Researchers agree that successful problem solving in mathematics requires coordinated application of a complex system of cognitive and metacognitive strategies and processes (Montague & Applegate, 1993). The cognitive-metacognitive framework underlying our study was developed by Montague and Bos (1986). This model identified seven cognitive processes (read, paraphrase, visualize, hypothesize, estimate, compute, check) and three metacognitive activities (self-instruction, self-questioning, self-monitoring) which are essential for effective problem solving. We used this framework in our study to analyse the spontaneous use of different representations and further elements of the problem solving process on complex story problems (i.e. "persistence", average time required; "total word problem score", number of correctly solved tasks).

The development of problem solving competences is an important subject of the curriculum for mathematics in German primary schools (Ministerium für Bildung, Frauen und Jugend, 2002). The TIMMS-study displayed that German students show a great deficit in problem solving competences concerning their abilities in mathematics (Baumert et al., 1997). Now, scientists have to find out where these deficits come from and what can be done to solve this matter. The underlying study is situated in the field of "problem solving" and made a contribution to the goal mentioned above.

In order to solve the complex story problems, students of this study were allowed to use different working materials. According to Schnotz, Baadte, Müller and Rasch (in press) these working materials

serve as external representations. A representation is an object or an occasion, which stands for something else (Schnotz, Baadte, Müller & Rasch, in press) and it is a very important element of the problem solving process in mathematics education. An adequate representation helps to find the solution more easily (Anderson, 1989), which has already been shown in various studies (Stern, 2005).

Neverless, studies on the use of different representations and further elements of the problem solving process on complex story problems are rare. By closing this gap in research, an effort is made to help students in their struggles with problem solving in mathematics at school (Baumert et al., 1997). Our study made a contribution to this goal.

In this context Ainsworth (2006) has worked intensely with external representations and developed a conceptual framework to use representations effectively (DeFT- Desing, Functions, Task). The DeFT-Framework and its remarks were an important basis to analyse the representations in this study.

Research Questions

The aim of this study was the examination and analysis of the spontaneous use of different representations and further elements of the problem solving process on complex story problems at primary schools. This included the following three research questions:

1) What kinds of representations do students of the two ability groups of 2^{nd} and 4^{th} grade use when working on complex story problems?

2) What are the differences in the persistence and total word problem score between the two ability groups of 2^{nd} and 4^{th} when working on complex story problems?

3) What are the specific differences of the research questions 1) and 2) between the two class levels?

Methods

A total number of 18 students of 2^{nd} and 20 students of 4^{th} grade from a public primary school participated in our study. The participants were divided into two groups, according to their results in a German standardized mathematics proficiency test. There was a group of higher and a group of lower maths competence. The tasks used in the study were selected from a book consisting of a collection of complex story problems (Rasch, 2008). The problems differ in the mathematical areas they are based on and the subject areas they deal with. A solution scheme was formed for each task based on the rational task analysis (Resnik & Ford, 1981). The schemes for the tasks of this study contained all possible ways to solve the tasks and appropriate essential steps. They were further differentiated according to possible kinds of representations. Moreover, the schemes contained possible comments the investigator could make, in order to help the students if they were not able to solve the problem on their own.

In our study, students were successively given five complex story problems. In up to 40 minutes, they could work on these tasks. They were given different working materials, (counting blocks and sticks, blank sheets and a pencil) which they were allowed to use at all times. If the students were not able to continue on their own, fixed clues from the investigators were used to help them.

While working on the story problems, the students were videotaped individually. The problem solving processes were analysed according to the solution schemes and a newly developed system of categories for this study. These devices of analysis complemented one another and together covered the entire problem solving process.

Preliminary Results

The results of our study show that the ability groups from both class levels differ in terms of representations they used and the time they needed for solving – this difference is also recognizable between the two class levels, in general. Students from the groups of "lower maths competence" showed a lower total word problem score, compared with the groups of "higher maths competence". Also, the students of 2nd grade showed a lower total word problem score than the students of 4th grade. The importance of other variables (i.e. mathematical and verbal skills) for the problem solving process are discussed in the course of this analysis.

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