Interpreting and Constructing Graphs: An analysis of 5th to 8th Grade Students' Difficulties

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Abstract. Two tasks were presented to 5th to 8th grade students. In the first one, participants answered a 12-item questionnaire on a bar graph. Questions of different levels of complexity were included. In the second one, a similar sample of students was asked to construct a bar graph from a set of listed data about the weights of a sample of girls and boys. Both studies included a follow up task with a small number of students. Our results showed a considerable low performance in both the interpretation and construction tasks. To explain this low performance, we identified the students' main cognitive difficulties: data crossing, frequency and the weight dissociation, comprehension of intervals of the measured variable, conceptual misconceptions and perceptive salience. Some of these difficulties appear in both graph processes: interpretation and construction.

Keywords: bar graphs; interpretation; construction; cognitive difficulties, 5th-8th graders

Introduction

From a psychological and educational perspective, two main processes related to graphic expertise have been identified: interpretation and construction. The first one is related to the ability to attribute meaning to data presented in a graph. The second one refers to the construction of a graph from data presented in another format (raw data, list, table). While most of the research in the field is devoted to graph interpretation (Friel, Curcio, & Bright, 2001; also see Leinhardt, Zaslavsky, & Stein, 1990 for a review), with much less attention devoted to processes of graph construction (Di Sessa, 2004; Nemirovsky, Tierney & Wright, 1998; Lehrer & Schauble), it is surprising that there are no studies that relate both processes.

Research on graph comprehension shows three different levels of processing graphic information: "read the data", "read between the data", and "read beyond the data") (Friel et al., 2001). "Read the data" implies extracting data from a graph (i.e., locating a given datum), in "read between the data, the student interpolates and finds relationships in the data as shown in a graph. Finally, "read beyond the data" implies extrapolating from the data and analyzing the relationships implicit in a graph (i.e., generating, predicting). In addition, a fourth level related with the understanding of the meaning of conventional aspects of graphs such as labels, colours and measures is proposed ("understanding basic information") (Marti et al., in press). To understand which aspects of graphs can explain low levels of understanding, different authors have identified frequent general misconceptions in interpreting graphs such as the confusion between interval/point and slope/height, the tendency to make iconic interpretations according to the shape of the graph or the great difficulty to understand the concept of variable (DiSessa, 2004; Leinhardt et al., 1990; Shah & Hoeffner, 2002). While most of this research concerns adult participants, the present study analyzes the level of graph understanding in Primary and Secondary students and how this competence changes with the educational level. Also, according with this first aim, it is necessary to understand the particular difficulties that students have when they are asked to interpret a graph.

Also, it is necessary to analyze the competence in constructing graphs, a process that has received less attention than interpretation, especially in Primary and Secondary education students. Constructing a graph is a complex task. Variables have to be clearly identified, data have to be organized into categories and the variables have to be graphically displayed in the coordinate system.

Although the interaction with graphic displays is quite common in school activities, middle school students show a low level of expertise in constructing graphs in general and also more concretely bar graphs (Lehrer & Schauble, 2000; Parmar & Singer, 2005; Wu & Krajcik, 2005). Along with Leinhard et al. (1990), the most difficult step in graph construction is setting up the axes and scales with the respective labels rather than plotting the points from a table. These authors (Lehrer & Schauble, 2000; Parmar & Singer, 2005; Wu & Krajcik, 2005) show that students' difficulties in constructing a bar graph were mainly associated with abstracting data by avoiding literality, removing redundant information and classifying the data by crossing two variables.

Thus, the present paper a focuses on middle school students' processes of graph interpretation and construction and especially their degree of similarity in order to focus on instruction.

Method

To address these goals we carried on two studies. Four hundred and sixty-five students were drawn from four levels of compulsory education (5^{th} grade, mean age=10;8 years; , 6^{th} grade, mean age=11;9 years; 7^{th} grade mean age=13;0 years and 8^{th} grade, mean age=14;1 years). In the first study (interpreting a graph), 205 participants were asked to answer a 12-item questionnaire about a bar graph. The bar graph represented the frequency distribution of girls and boys according to their weights. The items were designed to address different processing demands (basic questions, read data, interpolate data, and relate data with global understanding). In the second study, 260 students were asked to construct a graph from a list of data that included the names, ages and weights of a sample of boys and girls. The task demand was to construct a graph to show the number of girls and boys according to 4 weight intervals.

In order to understand the main difficulties students had when interpreting and constructing a bar graph, a small sample of students were individually interviewed in both studies. Our claim is that the follow up process of individually asking to interpret and construct the bar graph (by asking justifications and complementary data when students solved the problem) allowed a better understanding of the students' difficulties with bar graphs.

Results

The data of the interpretation task globally showed a low level of graph processing. Out of a total of 12 items, the overall mean of correct answers was 6.69 (standard deviation=2.21). The difference between degrees was significant ($\chi 2(3) = 9{,}438$, p=.024), while when each degree was compared with the others, only the results of 5th grade students were significatively different to the other grade means.

Only the "read data" questions and some "basic questions" were answered correctly by a high proportion of students. Questions that required a deep level of understanding (reading between and beyond data) had, in all cases, a percent of correct answers lower than 50%. It is interesting to point out that these kind of questions did not show any progress across school levels. Only some elementary questions ("read data questions") seemed to be easier to old students (6^{th}, 7^{th}, and 8^{th} degree) compared with 5^{th} degree students.

The qualitative analysis showed the main cognitive difficulties that students have in understanding the graph: the dissociation between the frequency and the weight variables, the comprehension of weight intervals, the wrong inferences caused by perceptive biases and the influence of conceptual misconceptions of the phenomenon represented in the graph.

The construction task also showed a low level of competence in elaborating a conventional bar graph. A high percent of students were able to graphically organize the data into a list format but failed to produce a frequency bar graph, showing a poor understating of the deep structure of the bar graphs. In general, the students did not succeed in making a graph with a correct format, that is representing frequencies in one of the axis (normally, the y-axis) and height ranges in the other (normally the x-axis). This difficulty with the format is related with the difficulty to organize and aggregate the data into frequencies. Instead of this, the students represented each individual in the x-axis and their height value in the y-axis, making the correspondence of "height" as the variable to be represented to "height" of the bar graph in the y-axis. The other difficulty, closely related to the former, was to cross the values of the two variables: gender (boys and girls) and the height variable (4 intervals).

Some of these difficulties appeared in both processes, interpretation and construction. For example, the distinction between frequency (number of subjects) and values of the variable that allow the identification of those subjects that belong to each category (each weight or each height interval) appear in both processes. In interpreting the graph, some students think that the heights of the bars in the graphic indicate weight instead of frequency. In the construction task, some students graphically represented each subject in the x-axis and the corresponding height in the y-axis, instead of aggregating all subjects belonging to one interval. In both cases, the same cognitive requirement (to abstract frequency from the measuring variable) seemed to be present.

Other difficulties appeared more specific to one task than the other. The requirement of crossing the values of the two variables seemed to be a difficulty in the construction task but not in the interpretation task. In the first case, the raw data had to be simultaneously categorized according to gender (boys/girls) and heights (four intervals) and this double categorization had to be represented in the graph (a bar for each subcategory). In the second case, students had to understand that some data (for example the height of a bar) were simultaneously related with gender and with a specific weight interval. According to our data, this "reading" process seemed easier than the data management to cross the two variables. Finally, some processes were present only in one of the tasks. For example, the perceptive bias (i.e. the students' judgements according to only a salient aspect of the graph) and the problem with intervals were related only to the interpretation task.

Our results address some educational questions. 1) The progress across compulsory education in interpreting and constructing a graph is very low and it only addresses the most basic competences. Therefore it is necessary to design instructional strategies to teach both competences in school. 2) The instructional strategies to help students to interpret and construct a graph must take into account the cognitive requirements involved in both processes.

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